Worksheet for Sections 7.6 and 8.1

- 1. Determine which of the following limits exist as a number, which as ∞ , and which don't exist at all. For any limit that exists as a number, determine the number.
 - (a) (i) $\lim_{x \to 0^+} x^x$ (ii) $\lim_{x \to 0^+} (1/x)^x$ (iii) $\lim_{x \to \infty} x^{\sin x}$
 - (b) (i) $\lim_{x \to 0^+} (1 1/x)^x$ (ii) $\lim_{x \to 0^+} (1 1/x)^{1/x}$ (iii) $\lim_{x \to \infty} (1 1/x)^x$
- 2. In the following, one integral in each pair can be evaluated by substitution and the other integral can be evaluated by integration by parts. In each case, write down which method applies, and evaluate the integral.
 - (a) i. $\int x \cos(x^2) dx$ (b) i. $\int \frac{x^2}{e^x} dx$ (c) i. $\int \frac{1}{x^4} \ln x dx$ ii. $\int x^2 \cos x dx$ ii. $\int xe^{(x^2)} dx$ ii. $\int \frac{1}{x} (\ln x)^4 dx$
- 3. Consider the integral $\int e^{\pi x} \sin(2x) dx$.
 - (a) In half of your group, solve the integral by performing integration by parts (twice!) with $u = \sin(2x)$ and $dv = e^{\pi x} dx$. In the other half of the group, solve the integral by performing integration by parts (twice!) with $u = e^{\pi x}$ and $dv = \sin(2x) dx$. Are the answers equivalent to each other? Explain in a sentence.
 - (b) Which of integration by parts processes is easier, or are they both about the same? Explain your answer. (Note: Sometimes it is not so simple to determine which part of the integrand is better as u, and which is better as dv.)
- 4. In each of the following, try to use the technique of integration by parts with the given u. In each, explain why that choice of u is not effective. Then write down a different function u and also write down dv that will make integration by parts effective, and tell why you chose your u and dv. (Don't perform the integration, however.)

a.
$$\int e^{-3x} x^{10} dx; \quad u = e^{-3x}$$
 b. $\int x^4 \sin x dx; \quad u = \sin x$